HEAT EXCHANGE BETWEEN PLASMA JET AND OBSTACLE UNDER UNSTEADY CONDITIONS

V. L. Sergeev and V. P. Veselov

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A method and the corresponding detectors for measuring large, varying heat fluxes have been studied. The heat fluxes from a plasma jet to a blunt object near the stagnation point have been measured under unsteady conditions.

For an arbitrary time variation of the heat flux at a surface, the temperature field of a semiinfinite object is given by the integral [1]

$$T(x,\tau) = T_0 + \frac{1}{\sqrt{\lambda c \rho \pi}} \int_0^{\tau} \frac{q(t)}{\sqrt{\tau - t}} e^{-x^2/4a(\tau - t)}\, dt. \qquad (1)$$

Partitioning the time interval under consideration into k subintervals, and assuming that the heat flux is constant over each such subinterval, we find the following equation for the heat flux [1]:

$$q_k = \Phi_{k,k}^{-1} \left\{ \frac{\lambda}{2x\sqrt{\Delta \mathrm{Fo}_x}} [T(x,\tau_k) - T_0] - \sum_{s=1}^{k-1} q_s \Phi_{k,s} \right\}, \qquad (2)$$

where

$$\Phi_{k,s} = \sqrt{k-s+1}\ \mathrm{ierfc} \frac{1}{2\sqrt{\Delta \mathrm{Fo}_x (k-s+1)}} - \sqrt{k-s}\ \mathrm{ierfc} \frac{1}{2\sqrt{\Delta \mathrm{Fo}_x (k-s)}}.$$

To determine the time dependence of the heat flux in Eq. (2) we must measure the temperature at some point of the object at a distance x from the surface. Calculations show that for measurement of heat fluxes on the order of 1 kW/cm^2 the solution in (2) is stable for $\Delta \mathrm{Fo}_x \geq 0.5$-1 [2]. When a thermocouple is placed 1-2 mm from the surface, the first value of the heat flux can be found at a time 0.005-0.01 sec after the beginning of the process. This time interval is the minimum value of the time intervals into which the curve $T = f(\tau)$ can be partitioned in order to find a stable solution.

The detectors used in this method are long copper rods with heat-insulated lateral surfaces. The temperature at the rear end of the rod is monitored by a thermocouple. Near the front end of the rod, between the rod and the insulation, is an annular contact 0.5-1 mm wide; over the remainder there is an air-filled gap.

Comparison of the method of a semiinfinite object with the method of [3] shows (see Fig. 1 of [4]) that the two give approximately the same results. To determine the heat fluxes by the second method it is necessary to measure the rod temperature at four points along its length. For a constant value of the heat flux, the method of a semiinfinite object agrees within the measurement error (10%-15%) with the exponential method for measuring heat fluxes (Fig. 2 of [4]). Finally, the test of the method reported below also confirms its suitability for measurements (Fig. 2).

To test the applicability of the one-dimensional theory [Eq. (2)] in the case of cylindrical detectors, we carried out a numerical calculation of the temperature fields for a detector model. We treated versions of the detector with textolite and copper protective sleeves. For version a (Fig. 1) the heat flux was assumed to be 0.7 or 3 kW/cm^2, corresponding to concrete experiments; for version b it was q = 0.55 kW/cm^2, and the width

A. V. Lykov Institute of Heat and Mass Transfer, Academy of Sciences of the Belorussian SSR, Minsk.
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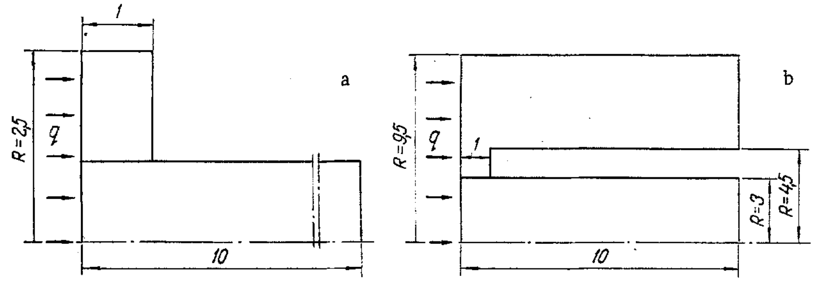

Fig. 1. Model of the detectors. a) Detector with textolite sleeve; b) copper sleeve.

of the contact between the detector and the sleeve was 1 or 0.4 mm. On the basis of preliminary calculations we chose a length of 10 mm for the detector; this is sufficient for simulation of a semiinfinite object over a time interval of 0.2-0.3 sec.

In accordance with the explicit calculation scheme (whose applicability for a problem of this type was checked beforehand through a comparison with the analytic solution for a semiinfinite object), the temperature at the interior points of the model detector (Fig. 1) was determined from

$$t_{i,m,k+1} = [1-2(\Delta \mathrm{Fo}_r + \Delta \mathrm{Fo}_z)]\, t_{i,m,k} + \Delta \mathrm{Fo}_r \left(1 - \frac{\Delta r}{2r_i}\right) t_{i-1,m,k} + \Delta \mathrm{Fo}_r \left(1 + \frac{\Delta r}{2r_i}\right) t_{i+1,m,k} + \Delta \mathrm{Fo}_z (t_{i,m-1,k} + t_{i,m+1,k}). \quad (3)$$

The step along the z direction was chosen to be 0.2 mm, and we chose $\Delta r = 0.5$ mm; then the step $\Delta\tau$ can be assumed to be 10^{-4} sec on the basis of the stability condition $0 \le [1 - 2(a\Delta\tau)/(\Delta r)^2 + a\Delta\tau/(\Delta z)^2] \le 1$. We assumed ideal thermal contact at the contact between the detector and the sleeve; at the other surfaces, except the front end, we assumed there was no heat transfer.

Analysis of the temperature fields found shows that in the detector there are radial temperature gradients, which have essentially no effect on the temperature at the axis of the detector in the cases considered (Fig. 2). Reconstruction of the heat flux on the basis of the temperature at the axis at a distance of 1 mm from the end, by means of the method of a semiinifinte object, yields values approximately the same as the original fluxes. In particular, for the case shown in Fig. 2 the heat flux in the numerical calculation was assumed to be 3 kW/cm^2, which is in good agreement with the calculated values of q (points 9). It also follows from this chart that there are no significant radial heat fluxes in the detectors used, so that they are suitable for measurements.

Measurement of the heat fluxes in the initial stage of the heating of a blunt object near the stagnation point in a plasma jet [2, 4] showed that the heat fluxes usually increase from zero to a steady-state value (Figs. 2 and 3), which is governed by the properties of the gas flow and by the shape of the object [5]:

$$q = 4.5 \cdot 10^{-4} R^{-0.5} p_0^{0.25} (p_0 - p_\infty)^{0.25} (h_0 - h_w). \quad (4)$$

In some cases the heat fluxes were constant, beginning with the first value found. The behavior of the heat flux does not depend on the nature of the gas [6] (air or argon) or the properties of the gas flow [6] (Figs. 2 and 3). The heat flux is the same if the detector is in a coaxial arrangement with the nozzle before the appearance of the plasma jet or if the detector is inserted into the jet after the apparatus has been turned on [6].

This analysis of the temperature fields in the detector showed that the observed increase in q during the initial stage cannot be attributed to heat fluxes across the contact between the detector and the protective sleeve. The change in the heat flux was caused by external factors.

The problem of the flow of a gas with constant properties around a blunt object, with unsteady heat exchange, can be formulated as follows [7]:

$$f''' + ff'' + \frac{1}{2}(1 - f'^2) = 0, \quad \frac{\partial\theta}{\partial\tau_*} = \mathrm{Pr}\, f \frac{\partial\theta}{\partial\eta} + \frac{\partial^2\theta}{\partial\eta^2}. \quad (5)$$

Neglecting the second term of the energy equation near the stagnation point, where the velocity components vanish, we reduce the heat-transfer problem to a heat-conduction problem in order to study the nature of the

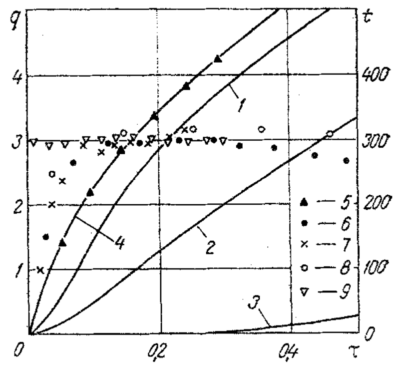

Fig. 2

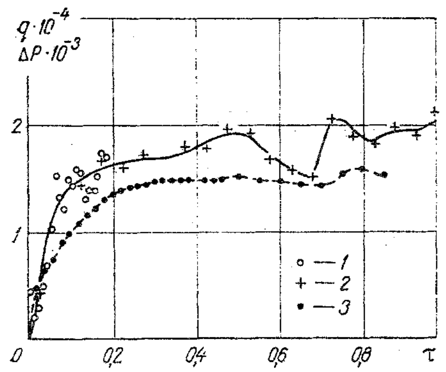

Fig. 3

Fig. 2. Characteristics of the heating of a detector with a spherical textolite protective sleeve by a plasma jet. $q_{st} = 3$ kW/cm^2; N = 147 kW, G = 7 g/sec (air). 1-5) Temperature at the axis at several distances from the front end; 1,4,5) 1 mm; 2) 5 mm; 3) 30 mm; 1,2,3) experimental; 4) calculated for a semiinfinite object with q = 3 kW/cm^2; 5) numerical calculation for a detector model with q = 3 kW/cm^2; 6-9) $q = f(\tau)$; 6) $\Delta\tau = 0.05$ sec with respect to curve 1; 7) $\Delta\tau = 0.025$ sec, curve 1; 8) $\Delta\tau = 0.1$ sec, curve 2; 9) $\Delta\tau = 0.025$ sec, curve 5. The temperature t is in degrees Celsius.

Fig. 3. Characteristics of the heating of a plane detector by a plasma jet immediately after the apparatus is turned on. N = 51 kW, G = 1.55 g/sec (nitrogen). The distance from the nozzle to the detector is L = 10 mm. 1) $q = f(\tau)$ with $\Delta\tau = 0.01$ sec; 2) 0.05 sec; 3) $\Delta p_k = f(\tau)$.

change in the heat flux at the surface. For arbitrary variations of the temperatures of the surface and the liquid at a distance δ from the surface, we find [8]

$$q = \frac{\lambda(\theta_{l,\tau} - \theta_{s,\tau})}{\sqrt{\pi a \tau}\, \mathrm{erf}\,(\delta/2\sqrt{a\tau})} \tag{6}$$

and

$$\alpha = \frac{q}{\theta_l - \theta_s} = \frac{\lambda}{\sqrt{\pi a \tau}\, \mathrm{erf}\,(\delta/2\sqrt{a\tau})} .$$

The first factor in the denominator increases as time elapses, while the second decreases. The heat-transfer coefficient and the heat flux can thus either increase or decrease as time elapses. An estimate of the behavior of q for the heat-transfer conditions prevailing in the plasma jet (the thickness δ is assumed to be $2.4\sqrt{\nu/2\beta}$ [9]) reveals a decrease of the heat flux as time elapses.

A decrease in heat flux over time is also predicted by a numerical solution [8] of the energy equation with the following temperature dependence for the gas properties:

$$\rho c_p \frac{\partial t}{\partial \tau} = \frac{\partial}{\partial y}\left(\lambda \frac{\partial t}{\partial y}\right) , \tag{7}$$

where $\rho = C/h$; $s = \int_0^t \lambda dt = A_1 + Ah$; $h = \int_0^t c_p dt$; C, A_1, and A are constants; and the temperature of the liquid changes in a step manner.

The scale time for the heating of the gas layer adjacent to the object was estimated on the basis of the heat-conduction problem for a plate in contact with a semiinfinite object [8]. The thickness of the plate simulating the boundary layer was determined from the equation given above [9]. The calculated results for velocities of 100 and 1000 m/sec are $0.7 \cdot 10^{-4}$ and $0.7 \cdot 10^{-5}$ sec, respectively. The scale time for the formation of the thermal boundary layer according to the data of [7] is $0.5 \cdot 10^{-4}$ or $0.5 \cdot 10^{-5}$ sec for these velocities; according to the data of [10], this time is $1.8 \cdot 10^{-4}$ or $1.8 \cdot 10^{-5}$ sec. These values are well below the experimental values of the time interval over which the heat flux changes, i.e., 0.1-0.2 sec (Figs. 2 and 3).

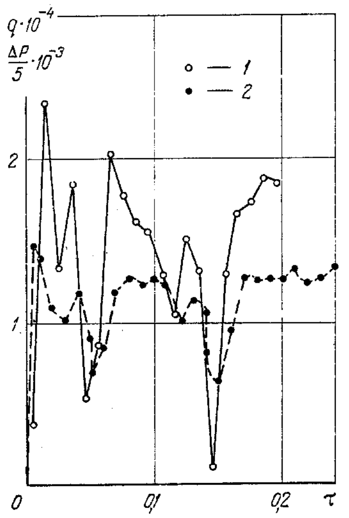

Fig. 4. Characteristics of the heating of a plane detector by a plasma jet with fluctuating parameters. N = 48 kW, G = 1.55 g/sec, L = 15 mm. 1) q = $f(\tau)$; 2) $\Delta p_k = f(\tau)$. q is in kilowatts per square meter; $\Delta P/5 \cdot 10^{-3}$ is in newtons per square meter, and τ is seconds.

The changes in the heat flux observed experimentally during the initial stage of the heating therefore cannot be explained on the basis of unsteady thermal processes in the boundary layer. In each case there can be particular factors governing the change in the heat flux.

The measurements showed that when the detector is inserted into the jet the scale time for the change in the heat flux is the same as the time corresponding to radial motion of the sensitive element in the jet. The position of the detector was determined with a loop oscillograph. When a shield was used over the time required to insert the detector and then removed in less than 0.01 sec, the rising part of the $q = f(\tau)$ curve vanished, and the heat flux was constant from the first value.

If the apparatus was turned on with the detector already in place, the heat flux "followed" the change in the pressure, which was not established instantaneously, according to measurements of the pressure in the discharge chamber (Fig. 3). This relationship was found not only in the initial stage of the heating but also upon the appearance of pressure fluctuations in the steady-state flow of the plasma jet (Fig. 4). The strain gauge used as a pressure gauge in these experiments was capable of measuring the pressure at a frequency of 200 Hz. Accordingly, in several cases the change in the heat flux can be attributed to a change in the stagnation pressure.

To compare the heat fluxes under steady and unsteady conditions, we carried out experiments in which we measured q while the heat-flux detector was moved along the plasma jet. We first used the exponential method to measure the steady-state heat fluxes at various distances from the end of the nozzle. The heat fluxes were found to fall off with increasing distance from the nozzle; the change in the steady-state heat flux during the motion of the detector was 2 kW/cm^2, while the velocity was 1.6 m/sec. From these values we conclude that there was a change in the heat flux to the detector at a rate of about 55 (kW/cm^2)/sec. This change was due to changes in the enthalpy, the pressure, and the velocity gradient near the stagnation point along the jet.

Measurements of the heat flux with this detector under these conditions of unsteady heat fluxes showed that the heat fluxes found under steady and unsteady conditions agree well. Accordingly, the processes leading to the restructuring of the boundary layer do not substantially affect the measurements of the heat fluxes for the rates of change of the external conditions prevailing in the present experiments.

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DIFFERENTIAL FORM OF THE UNIVERSAL EQUATION OF THE LAMINAR BOUNDARY LAYER

N. G. Khislavskaya

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We propose a new approach to composing a universal equation of the laminar boundary layer in generalized similarity variables.

§1. The wide use of electronic digital calculators has greatly reduced interest in approximate methods of ; of computation. However, the problem of establishing general rules to describe the effect of factors external to the boundary layer (such as the velocity distribution at the outer boundary, blowing or suction velocities, body surface temperatures, external magnetic field stresses, etc.) on terminal characteristics (friction stress, heat-transfer coefficient, flow separation location, etc.) continues to be one of practical and fundamental significance. These rules express general tendencies of various processes such as flow drag, heat transfer, and related motions in boundary layers.

The "generalized similarity method," proposed in [1] by Loitsyanskii, makes it possible to examine broad classes of problems of boundary-layer theory by transforming the boundary-layer equation to a "universal" generalized-similarity form requiring only a single numerical integration. The resulting tables of solutions, prepared once and for all, express general rules and relationships among the basic characteristics of the boundary layer.

§2. In its initial form the generalized similarity method was first published in [1]. Its distinguishing feature was that its basic universal equation was of integrodifferential form, in which differential and integral functionals of the unknown solution were present. In the rather simple cases treated in that paper only minor complications were encountered in numerically integrating the fundamental equation. Results of the integration and a corresponding bibliography can be found in [2].

The attempt to apply the method to more involved cases (nonstationary boundary layer, jets and wakes in arbitrary pressure fields, etc.) showed that by reducing the universal equations to purely differential form one could, in spite of the introduction thereby of an increase in the number of independent variables, significantly simplify the form of the universal equation and aid in effecting the first stage of the method, namely, that of deriving general rules. In the present paper we develop the basic notion involved in the transition of the universal equation from an integrodifferential form to one of purely differential form, and we apply it to a physically realistic and sufficiently general example of a two-dimensional high-speed boundary layer in a homogeneous incompressible fluid; generalization of the notion to more involved motions is then perfectly straightforward.

§3. As a direct substitution of an affinely similar form of the stream function $\psi = U\delta\psi_1(y/\delta)$ into the general Prandtl equation reveals, there appears in the equation a particular pair of conjugate parameters: $f_1 = U'z$, $\overline{f_1} = Uz'$ which are explicit functions of x [U(x) is the speed at the outer edge of the boundary layer; $z = \delta^2/\nu$; δ is a conditional "thickness" of the boundary layer; and the primes indicate differentiation with respect to x], thereby violating the universality of the new form of the equation. Introduction of these parameters into a number of the independent [by virtue of the arbitrariness of U(x)] variables, i.e., a transition

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